PHY180 Unit 12

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Recall uniform circular motion:

We have some circlular motion with an angular velocity of ω, it has radius $\overrightarrow{r}$, with an angle of θ to the x axis.

$$x = |\overrightarrow{r}|\, cos\theta \rightarrow x = Acos(\omega t + \phi)$$

$$y = |\overrightarrow{r}|\, sin\theta \rightarrow x = Asin(\omega t + \phi)$$

$$\theta = \omega t + \phi$$

$$A = |\overrightarrow{r}|$$

What force ensures circular motion?

$$\overrightarrow{F} = m\overrightarrow{a}$$

Uniform circular motion:

$$\frac{dw}{dt} = 0\alpha$$

$\rightarrow$ only radial component of $\overrightarrow{F}$

$$a_c = r\omega^2$$

$$x(t) = rcos(\omega t)$$

$$\frac{d^2x}{dt^2} = -rcos(\omega t)\omega^2$$

$$= -\omega^2 x(t)$$

$$F_x = -m\omega^2 x(t)$$

Oscillator: Spring

$$F_x = -kx \text{ (spring acting on a mass)}$$

$$F_x = ma_x$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x(t)}{dt^2} + \frac{k}{m}x(t) = 0$$

Educated guess at solution:

$$x(t) = Acos(\omega t + \phi_i)$$

$$\frac{d^2x(t)}{dt^2} = -A\omega^2 cos(\omega t + \phi_i)$$

Solution IFF:

$$\rightarrow \omega^2 = \frac{k}{m}$$

$$\rightarrow \omega = \sqrt{\frac{k}{m}}$$

Find expression for speed/acceleration:

$$x(t) = Acos(\omega t + \phi_i)$$

$$v(t) = -A\omega sin(\omega t + \phi_i)$$

$$a(t) = -A\omega^2 cos(\omega t + \phi_i)$$

$$= -\omega^2 x(t)$$